## SUMMER WORKSHOP IN MATHEMATICS

## (SWIM@KSOM - 2025)

## Introduction to University Mathematics

(Problem Sheet 2)

1. Ordered Field: A order field F is an ordered set such that for all  $x, y, z \in F$ 

1. x + y < x + z whenever y < z

2. xy > 0 whenever x > 0, y > 0

Prove that the following are true in an ordered field.

- (a)  $x > 0 \implies -x < 0$
- (b) xy < xz whenver x > 0 and y < z
- (c) xy > xz whenver x > 0 and y > z
- (d)  $x^2 > 0$  for all  $x \neq 0$
- (e) 1 > 0
- (f)  $0 < \frac{1}{y} < \frac{1}{x}$  whenever 0 < x < y

(g) 
$$x < \frac{x+y}{2} < y$$

- 2. Prove that the field of complex numbers cannot be an ordered field.
- 3. Prove that  $\inf S = -\sup\{-s : s \in S\}$
- 4. Prove that  $\sup(A \cup B) = \max\{\sup A, \sup B\}$
- 5. Suppose S is an ordered set with supremum property, then prove that S has infimum property also.
- 6. Archemedian Property: Prove that Natural Numbers are unbounded.
- 7. Density Property: Prove that between any two real numbers there is a rational number.
- 8. Prove that between any two real numbers there is an irrational number.