## SUMMER WORKSHOP IN MATHEMATICS

## (SWIM@KSOM - 2025)

## Introduction to University Mathematics

(Problem Sheet 1)

1. Consider the infinite sum

$$S = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$$

Find S (Hint: Compute 2S). Do the same technique for the series

 $1+2+3+\ldots$ 

Is everything fine!!!

2. (Interchanging Limits) Can we interchange limit always as below (provided the limit exist)?

$$\lim_{x \to 0} \lim_{y \to 0} f(x, y) = \lim_{y \to 0} \lim_{x \to 0} f(x, y)$$

3. (Limit and Integral) Do the following equality holds always?

$$\lim_{n \to \infty} \int f_n = \int \lim_{n \to \infty} f_n$$

- 4. Prove the following properties addition of natural numbers (defined through Peano's axioms).
  - (a) n + 0 = n = 0
  - (b) n + s(m) = s(n+m)
  - (c) Commutativity: n + m = m + n
  - (d) Associativity: (a + b) + c = a + (b + c)
- 5. Prove the properties of ordering of natural numbers:
  - (a) Reflexive:  $a \ge a$  for all  $a \in \mathbb{N}$
  - (b) Antisymmetric:  $a \ge b$  and  $b \ge a$  implies a = b for all  $a, b \in \mathbb{N}$
  - (c) Transitive:  $a \ge b$  and  $b \ge c$  implies  $a \ge c$  for all  $a, b, c \in \mathbb{N}$
  - (d)  $a \ge b$  then  $a + c \ge b + c$  for all  $a, b, c \in \mathbb{N}$