SUMMER WORKSHOP IN MATHEMATICS

(SWIM@KSOM - 2025)

Analysis

(Problem Sheet 2)

- 1. Does there exist $\alpha, \beta \in \mathbb{R} \setminus \mathbb{Q}$ such that $\alpha^{\beta} \in \mathbb{Q}$
- 2. Prove that convergent sequences of real numbers is bounded.
- 3. Suppose that $a_n \to 0$ and $|x_n x| \le a_n$, then show that $x_n \to x$.
- 4. Evaluate the following limits and prove the convergence:

(a)
$$\sqrt{4n^2 + n - 2n}$$

(b) $\lim_{n \to \infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$
(c) $\lim_{n \to \infty} \frac{n^2}{n!}$

- 5. Squeeze Theorem: Suppose $\{x_n\}, \{y_n\}, \{z_n\}$ be sequences of real numbers such that $x_n \leq y_n \leq z_n$ for all $n \in \mathbb{N}$ and $\lim_{n\to\infty} x_n = \lim_{n\to\infty} z_n = a$. Prove that $\lim_{n\to\infty} y_n = a$.
- 6. Evaluate the following limits:
 - (a) $\frac{\sin n}{n}$
 - (b) $\frac{\cos n}{n^2}$
- 7. Consider the sequence $\{x_n\}$ defined as $x_1 = 0, x_2 = 1$ and

$$x_{n+2} = \frac{x_{n+1} + x_n}{2}.$$

Evaluate $\lim_{n\to\infty} x_n$.