SUMMER WORKSHOP IN MATHEMATICS

(SWIM@KSOM - 2025)

Analysis

(Problem Sheet 1)

---

1. Prove that limit of a sequence if exist is unique.
2. Suppose $a_n \to a$ and $a_n \geq 0$ for all n. Prove that $a \geq 0$.
3. Suppose $x_n \to x$ and $y_n \to y$ such that $x_n \leq y_n$ for all n. Prove that $x \leq y$.
4. Evaluate the following limits and prove the convergence:
   (a) $\lim_{n\to\infty} \frac{3n+2}{n+1}$
   (b) $\lim_{n\to\infty} \frac{5n+2}{2n-1}$
   (c) $\lim_{n\to\infty} \frac{1}{\sqrt{n}}$
   (d) $\lim_{n\to\infty} (\sqrt{n+1} - \sqrt{n})$
5. Suppose $x_n \to x$ and $y_n \to y$ prove that
   (a) $x_n \pm y_n \to x \pm y$
   (b) $x_n y_n \to xy$
   (c) If $y_n, y \neq 0$, then $\frac{x_n}{y_n} \to \frac{x}{y}$
6. Consider the sequence $\{x_n\}$ defined as $x_1 = 0, x_2 = 1$ and

   $$x_{n+2} = \frac{x_{n+1} + x_n}{2}.$$

   Write first five terms of the sequence.