SUMMER WORKSHOP IN MATHEMATICS

(SWIM@KSOM - 2025)

Algebra

(Problem Sheet 1)

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1. Identify the group $9\mathbb{Z} + 12\mathbb{Z}$.
2. Prove that intersection of subgroups is a subgroup. What about union?
3. What is the subgroup formed by intersection of $n\mathbb{Z} \cap m\mathbb{Z}$.
4. Find a finite subgroup of non-zero reals with multiplication as the group operation.
5. Find a group of order 3 of non-zero complex numbers with multiplication as the group operation.
6. Find a group of order n of non-zero complex numbers with multiplication as the group operation.
7. Define subgroup generated by a set S. What is the subgroup generated by $[0, 1]$ in $\mathbb{R}$ with respect to addition?
8. Prove any finitely generated subgroup of $\mathbb{Q}$ is cyclic.